VIVEK TUTORIALS

X (English)

(Special Test)

Mathematics Part - II-(6)

DATE: 21-02-19

TIME: 40 Mins

MARKS: 30

SEAT NO:

Q.1 Multiple Choice Questions 2

1 If the initial arm rotates 70° in clockwise direction, then in which quadrant will the terminal arm lie ?

a. I b. III c. IV d. II

Ans Option c

2 What is the value of cosec 45°?

a. $\frac{1}{\sqrt{2}}$ b. $\sqrt{2}$ c. $\frac{\sqrt{3}}{2}$ d. $\frac{2}{\sqrt{3}}$

Ans Option b

Q.2 Solve the following 4

1 Prove the following

$\cos^2\theta\,(1 + \tan^2\theta) = 1$

Ans LHS $= \cos^2\theta\,(1 + \tan^2\theta)$

$= \cos^2\theta \times \sec^2\theta$ ...[$1 + \tan^2\theta = \sec^2\theta$]

$= \cos^2\theta \times \frac{1}{\cos^2\theta}$ ...[$\sec\theta = \frac{1}{\cos\theta}$]

$= 1$

$\therefore$ LHS = RHS

$\therefore$ $\cos^2\theta\,(1 + \tan^2\theta) = 1$

2 If $\sin\theta = \frac{15}{17}$, find the value of $\cos\theta$, (θ is an acute angle)

Ans θ is an acute angle.

$\therefore$ all trigonometric ratios are positive

$\therefore$ $\sin^2\theta + \cos^2\theta = 1$ ... (Trigonometric identity)

$\therefore$ $\left(\frac{15}{17}\right)^2 + \cos^2\theta = 1$

$\therefore$ $\cos^2\theta = 1 - \left(\frac{15}{17}\right)^2 = 1 - \frac{225}{289}$

$\therefore$ $\cos^2\theta = \frac{289 - 225}{289} = \frac{64}{289}$

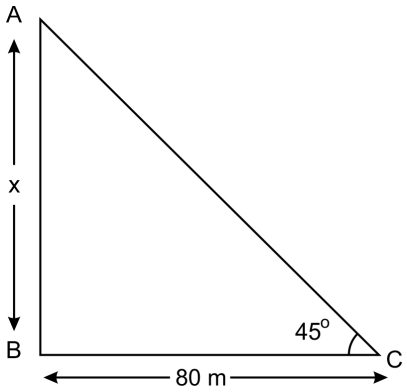
$\therefore$ $\cos\theta = \frac{8}{17}$ ... (Taking square root of both the sides)

The value of $\cos\theta$ is $\frac{8}{17}$.

Q.3 Answer the following 4

1 For a person standing at a distance from a church the angle of the elevation of its top is measure 45º. Find the height of the church.

Ans



Seg AB represent the church C re-present the position where the person is standing. Distance of the man from the church = BC = 80 m

Angle of deviation = ∠ACB = 45

Let the height of the church = AB = X m.

In right angled Δ ABC,

$\tan 45^\circ = \frac{AB}{BC}$

$1 = \frac{X}{80}$ (standard Trigonometric ratio)

$\therefore \quad X = 80 \times 1$

$\therefore \quad X = 80$ m

Ans. The height of the church is 80 m.

2 If $\tan\theta = \frac{3}{4}$ then find the value of $\sec\theta$.

Ans If $\tan\theta = \frac{3}{4}$

$1 + \tan^2\theta = \sec^2\theta$

$\therefore \quad 1 + \left(\frac{3}{4}\right)^2 = \sec^2\theta$

$\therefore \quad 1 + \frac{9}{16} = \sec^2\theta$

$\therefore \quad \frac{25}{16} = \sec^2\theta$

$\therefore \quad \sec\theta = \frac{5}{4}$

Q.4 Solve the following 6

1 Prove the following.

$(\sec\theta + \tan\theta)(1 - \sin\theta) = \cos\theta$

Ans LHS = $(\sec\theta + \tan\theta)(1 - \sin\theta)$

$= \left(\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}\right)(1 - \sin\theta) \quad \ldots \left[\tan\theta = \frac{\sin\theta}{\cos\theta}\right]$

$= \left(\frac{1 + \sin\theta}{\cos\theta}\right)(1 - \sin\theta)$

$= \frac{(1 + \sin\theta)}{\cos\theta}(1 - \sin\theta)$

$= \frac{1 - \sin^2\theta}{\cos\theta} \quad \ldots [(a + b)(a - b) = a^2 - b^2]$

$= \frac{\cos^2\theta}{\cos\theta} \quad \ldots [\sin^2\theta + \cos^2\theta = 1]$

$= \cos\theta$

$\therefore$ LHS = RHS

∴ $(\sec\theta + \tan\theta)(1 - \sin\theta) = \cos\theta$

2 Prove the following.

$\sec\theta (1 - \sin\theta)(\sec\theta + \tan\theta) = 1$

Ans LHS = $\sec\theta (1 - \sin\theta)(\sec\theta + \tan\theta)$

$= (\sec\theta - \sec\theta \times \sin\theta)(\sec\theta + \tan\theta)$

$= \left(\sec\theta - \frac{1}{\cos\theta} \times \sin\theta\right)(\sec\theta + \tan\theta)$

$= \left(\sec\theta - \frac{\sin\theta}{\cos\theta}\right)(\sec\theta + \tan\theta)$

$= (\sec\theta - \tan\theta)(\sec\theta + \tan\theta)$ ... $\left[\tan\theta = \frac{\sin\theta}{\cos\theta}\right]$

$= \sec^2\theta - \tan^2\theta$ ... $[(a + b)(a - b) = a^2 - b^2]$

$= 1$ ... $[1 + \tan^2\theta = \sec^2\theta]$

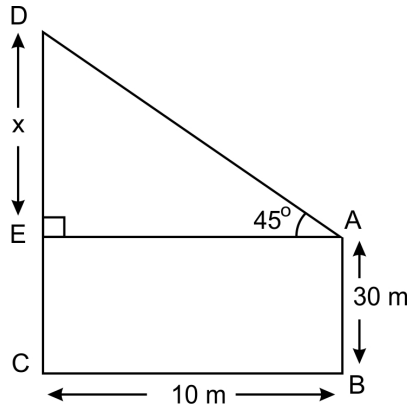
∴ LHS = RHS

∴ $\sec\theta (1 - \sin\theta)(\sec\theta + \tan\theta) = 1$

Q.5 Answer the following 8

1 Two building are in front of each other on either side of a road of width 10 meters. From the top of the first building, which is of 30 meters of height the angle of elevation of the top second is 45º what is the height of the building ?

Ans



Seg AB : Represent one building.

Seg CD : Represent other building.

Seg BC : Represent the road.

Height of the one building = AB = 30 m

Width of the road = BC = 10 m

The angle of elevation = ∠DAE = 45º

seg AE ⊥ seg CD

Leg DE = x m.

□ ABCE is a rectangle

∴ AB = CE = 30 m. [Opposite sides of a rectangle are]

and BC = AE = 10 m

In right angled ΔAED,

$\tan 45^\circ = \frac{DE}{AE}$

$1 = \frac{X}{10}$ (S.T.R.)

∴ x = 10 m.

∴ DE = 10 m.

∴ CD = CE + DE (C – E – D)

$= 30 + 10$

$\therefore$ CD = 40 m

Height of the second building is 40 m.

2 $\dfrac{\sqrt{\text{cosec X} - 1}}{\text{cosec X} + 1} = \dfrac{1}{\text{sec X} + \tan \text{X}}$

Ans $\sqrt{\dfrac{\text{cosec x} - 1}{\text{cosec x} + 1}} = \sqrt{\dfrac{(\text{cosec X} - 1) \times (\text{cosec X} - 1)}{(\text{cosec X} + 1) \times (\text{cosec X} - 1)}}$

$= \sqrt{\dfrac{(\text{cosec X} - 1)^2}{\text{cosec}^2 \text{X} - 1}}$

$= \sqrt{\dfrac{(\text{cosec X} - 1)^2}{\cot^2 \text{X}}}$

$= \dfrac{\text{cosec X} - 1}{\cot \text{X}}$

$= \dfrac{\text{cosec X}}{\cot \text{X}} - \dfrac{1}{\cot \text{X}}$

$= \dfrac{\dfrac{1}{\sin \text{x}}}{\dfrac{\cos \text{x}}{\sin \text{x}}} - \tan \text{x}$

$= \dfrac{1}{\cos \text{X}} - \tan \text{X}$

$= \text{sec X} - \tan \text{X}$

$= \dfrac{(\text{sec X} - \tan \text{X})\ (\text{sec X} + \tan \text{X})}{(\text{sec X} + \tan \text{X})}$

Multiplying & Dividing by (sec X + tan X)

$= \dfrac{\text{sec}^2 \text{x} - \tan^2 \text{x}}{\text{sec x} + \tan \text{x}}$

$= \dfrac{1}{\text{sec X} + \tan \text{X}}$

$\therefore \sqrt{\dfrac{\text{cosec X} - 1}{\text{cosec X} + 1}} = \dfrac{1}{\text{sec x} + \tan \text{x}}$

Q.6 Answer the following 6

1 $\dfrac{1 + \sin \text{A}}{\cos \text{A}} = \dfrac{1 + \sin \text{A} + \cos \text{A}}{1 + \cos \text{A} - \sin \text{A}}$

Ans $\sin^2 \text{A} + \cos^2 \text{A} = 1$ [Trigonometrical Identity]

$\therefore \cos^2 \text{A} = 1 - \sin^2 \text{A}$

$\therefore$ (cos A) (cos A) = (1 + sin A) (1 - sin A)

$\therefore \dfrac{\cos \text{A}}{1 - \sin \text{A}} = \dfrac{1 + \sin \text{A}}{\cos \text{A}}$

Let $\dfrac{\cos \text{A}}{1 - \sin \text{A}} = \dfrac{1 + \sin \text{A}}{\cos \text{A}} = \text{K}$ ... (i)

[Where k is non zero real constant]

$\therefore \text{K} = \dfrac{\cos \text{A} + 1 \sin \text{A}}{1 - \sin \text{A} + \cos \text{A}}$ [By Theorem on equal ratio]

$\text{K} = \dfrac{1 + \sin \text{A} + \cos \text{A}}{1 + \cos \text{A} - \sin \text{A}}$ ... (ii)

From (i) & (ii)

$\dfrac{1 + \sin \text{A}}{\cos \text{A}} = \dfrac{1 + \sin \text{A} + \cos \text{A}}{1 + \cos \text{A} - \sin \text{A}}$

2 Prove the following $\dfrac{\sin \theta - \cos \theta + 1}{\sin \theta + \cos \theta - 1} = \dfrac{1}{\sec \theta - \tan \theta}$

Ans LHS = $1 - \sin^2\theta = \cos^2\theta$

$\therefore \quad (1 + \sin\theta)(1 - \sin\theta) = \cos\theta \cdot \cos\theta$

$\therefore \quad \dfrac{1 + \sin\theta}{\cos\theta} = \dfrac{\cos\theta}{1 - \sin\theta}$ ... (I)

By theorem on equal ratios

Each ratio of (I) $= \dfrac{1 + \sin\theta - \cos\theta}{\cos\theta - (1 - \sin\theta)}$

$= \dfrac{1 + \sin\theta - \cos\theta}{\cos\theta - 1 + \sin\theta}$

$= \dfrac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1}$

$\therefore \quad \dfrac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \dfrac{\cos\theta}{1 - \sin\theta}$

$= \dfrac{\frac{\cos\theta}{\cos\theta}}{\frac{1 - \sin\theta}{\cos\theta}}$ [Dividing both numerator and denominator by $\cos\theta$]

$= \dfrac{1}{\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}}$

$= \dfrac{1}{\sec\theta - \tan\theta}$

$\therefore$ LHS = RHS